Precalc Warm Up # 11-1

1. Find the sum:

$$2 + -8 + 32 + -128 + \ldots + 2{,}097{,}152$$

Write the Partial Fraction Decomposition for:

2. $\dfrac{7x - 18}{x^2 - 3x - 4}$

3. $\dfrac{4x^2 + 13x + 12}{x^3 + 4x^2 + 4x}$

HW Questions: p.259

EXERCISES 8.2.2

1. Find the common ratios of these geometric sequences:

(d) $9, -3, 1, -\frac{1}{3}, \frac{1}{9}, \ldots$

2. Find the term indicated for each of these geometric sequences:

(c) $9, -6, 4, -\frac{8}{3}, \ldots$ 9th term.

3. Find the number of terms in each of these geometric sequences and the sum of the numbers in each sequence:

(a) $4, 12, 36, \ldots, 236196$

(c) $100, -10, 1, \ldots, -10^{-5}$

$$S_8 = \frac{100(1-(-0.1)^8)}{1-(-0.1)}$$

$r = -\frac{1}{10}$

$$\frac{-10^{-5}}{10^2} = \frac{\cancel{10^2}\left(-\frac{1}{10}\right)^{n-1}}{\cancel{10^2}}$$

$-10^{-7} = (-10^{-1})^{n-1}$

$-10^{-7} = -10^{1-n}$

$-7 = 1-n$

$n=8$

4. Write the following in expanded form and evaluate.

$n=7 \quad r=\frac{1}{2} \quad S_7=\frac{\frac{1}{2}(1-(\frac{1}{2})^7)}{1-\frac{1}{2}}$

$g_1=\frac{1}{2}$

$g_2=\frac{1}{4}$

(a) $\sum_{k=1}^{7}\left(\frac{1}{2}\right)^k$ (b) $\sum_{i=1}^{6} 2^{i-4}$ (c) $\sum_{j=1}^{4}\left(\frac{2}{3}\right)^j$

(d) $\sum_{s=1}^{4}(-3)^s$ (e) $\sum_{t=1}^{6} 2^{-t}$

b) $\sum_{i=1}^{6} 2^{i-4} = \boxed{S_6 = \frac{\frac{1}{8}(1-(2)^6)}{1-2}}$

$n=6$

$u_1=2^{-3}=\frac{1}{8}$

$u_2=2^{-2}=\frac{1}{4}$

$u_3=2^{-1}=\frac{1}{2}$

$r=2$

$= -\frac{(1-64)}{8}$

$= \frac{63}{8}$

4d) $\sum_{s=1}^{4}(-3)^s = S_4 = \frac{-3(1-(-3)^4)}{1-(-3)}$

$\boxed{n=4}$

$u_1=-3$

$u_2=9$

$u_3=-27$

$= -\frac{3(1-81)}{4}$

$= -\frac{3(-80)}{4}$

$=$

$-3+9+(-27)$

$r=-3$

5. The third term of a geometric sequence is 36 and the tenth term is 78,732. Find the first term in the sequence and the sum of these terms.

$g_3 = 36$ $\quad g_{10} = 78{,}732$

Use $g_n = g_1 \cdot r^{n-1}$

① $36 = g_1 \cdot r^2$

② $78{,}732 = g_1 \cdot r^9$

Use elimination to solve system:

$\frac{②}{①} \quad \frac{78732}{36} = \frac{\cancel{g_1} \cdot r^9}{\cancel{g_1} \cdot r^2}$

$\sqrt[7]{2187} = \sqrt[7]{r^7}$

$r = 3$

Now sub that in to find g_1

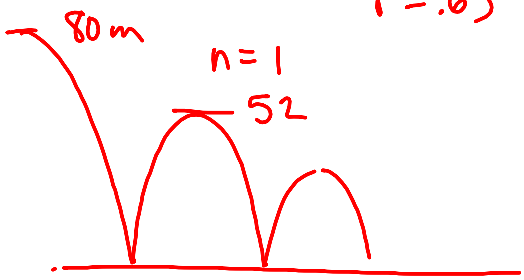
6. A bank account offers 9% interest compounded annually. If \$750 is invested in this account, find the amount in the account at the end of the twelfth year.

$A = P(1+r)^t$ where $r =$ interest rate

$= 750(1.09)^{12}$

7. When a ball is dropped onto a flat floor, it bounces to 65% of the height from which it was dropped. If the ball is dropped from 80 cm, find the height of the fifth bounce.

$r = .65$ $.65(80)$



$$g_n = g_1 r^{n-1}$$

$$g_5 = 52(0.65)^{5-1}$$

8. A computer loses 30% of its value each year.

(a) Write a formula for the value of the computer after n years.

(b) How many years will it be before the value of the computer falls below 10% of its original value?

Initial value

$100\% - 30\% = 70\%$

so $r = 0.7$

$$V_n = V_0(0.7)^n$$

b) $V_n < 0.10(V_0)$?

$$V_0(0.7)^n < 0.1(V_0)$$

$$\frac{\cancel{V_0}(0.7)^n}{\cancel{V_0}} < \frac{0.1(\cancel{V_0})}{\cancel{V_0}}$$

$$0.7^n < 0.1$$

$$\ln(0.7)^n < \ln 0.1$$

$$\frac{n(\ln 0.7)}{\ln 0.7} < \frac{\ln 0.1}{\ln 0.7}$$

$\ln 0.7 \approx -0.36$

$$n \gtrapprox 6.46$$

so after 7 years

9. A geometric sequence has a first term of 7 and a common ratio of 1.1. How many terms must be taken before the value of the term exceeds 1000?

n ?

$g_1 = 7 \qquad r = 1.1$

$$g_n > 1000$$

$$7(1.1)^{n-1} > 1000$$

$$(1.1)^{n-1} > \frac{1000}{7}$$

$$\ln(1.1)^{(n-1)} > \ln\frac{1000}{7}$$

$$\frac{(n-1)\cancel{\ln(1.1)}}{\cancel{\ln(1.1)}} > \frac{\ln\frac{1000}{7}}{\ln(1.1)}$$

$$n > \frac{\ln\frac{1000}{7}}{\ln 1.1} + 1$$

$$n \gtrapprox 53.06$$

$$\boxed{54 \text{ terms}}$$

* $\ln 1.1$ is not negative, Don't need to change the inequality

10. A colony of algae increases in size by 15% per week. If 10 grams of the algae are placed in a lake, find the weight of algae that will be present in the lake after 12 weeks. The lake will be considered 'seriously polluted' when there is in excess of 10000 grams of algae in the lake. How long will it be before the lake becomes seriously polluted?

$A_0 = 10$ gr. $\qquad r = 1.15 \qquad n = \#$ of weeks

$$A_n = 10(1.15)^n$$

$$A_{12} = 10(1.15)^{12}$$

$$\approx 53.5 \text{ gr.}$$

$$10(1.15)^n > 10{,}000$$

$$(1.15)^n > 1{,}000$$

$$\frac{n\cancel{\ln(1.15)}}{\cancel{\ln(1.15)}} > \frac{\ln 1{,}000}{\ln(1.15)}$$

$$n \gtrapprox 49.43$$

By the 50^{th} week

11. A geometric series has nine terms, a common ratio of 2 and a sum of 3577. Find the first term.

$n = 9 \quad r = 2 \quad S_9 = 3577$

use: $S_n = \dfrac{g_1(1-r^n)}{1-r}$

$3577 = \dfrac{g_1(1-2^9)}{1-2}$

$\dfrac{-3577}{(1-2^9)} = \dfrac{g_1\cancel{(1-2^9)}}{\cancel{(1-2^9)}}$

$g_1 = \dfrac{-3577}{1-2^9}$

$g_1 = 7$

12. A geometric series has a third term of 12, a common ratio of $-\frac{1}{2}$ and a sum of $32\frac{1}{16}$. Find the number of terms (n) in the series.

$g_3 = 12 \quad r = -\frac{1}{2}$

$g_1\left(-\frac{1}{2}\right)^2 = 12$

$g_1\left(\frac{1}{4}\right) = 12$

$g_1 = 48$

$S_n = \frac{513}{16}$

$\frac{513}{16} = \dfrac{48\left(1-\left(-\frac{1}{2}\right)^n\right)}{1-\left(-\frac{1}{2}\right)}$

$\frac{513}{16} = \dfrac{48\left(1-\left(-\frac{1}{2}\right)^n\right)}{\frac{3}{2}}$

$\frac{513}{16} = 32\left(1-\left(-\frac{1}{2}\right)^n\right)$

$\frac{513}{512} = 1-\left(-\frac{1}{2}\right)^n$

$\frac{1}{512} = -\left(-\frac{1}{2}\right)^n$

$\left(-\frac{1}{2}\right)^n = \left(-\frac{1}{2}\right)^9$

$\boxed{n = 9}$

14. A geometric series has a third term of 300, and a sixth term of 37500. Find the common ratio and the sum of the first fourteen terms (in scientific form correct to two significant figures).

$g_3 = 300 \qquad g_6 = 37500$

$g_1 r^2 = 300 \qquad \dfrac{g_1 r^5}{g_1 r^2} = \dfrac{37500}{300}$

$g_1 (5)^2 = 300 \qquad r^3 = 125$

$g_1 = 12 \qquad \boxed{r = 5}$

$$S_{14} = \frac{12(1 - 5^{14})}{1 - 5}$$

$$S_{14} = -3(1 - 5^{14})$$

$$\approx 1.8 \times 10^{10}$$

16. As a prize for inventing the game of chess, its originator is said to have asked for one grain of wheat to be placed on the first square of the board, 2 on the second, 4 on the third, 8 on the fourth and so on until each of the 64 squares had been covered. How much wheat would have been the prize?

$1, 2, 4, 8, \ldots \qquad n = 64$

$$S_{64} = \frac{1(1 - 2^{64})}{1 - 2}$$

$$\approx 1.84 \times 10^{19}$$

Today: Mixed AP's and GP's
Review tools from day 1:

To solve an equation with the variable in the exponent, isolate the term with the exponent then take the log (or ln) of both sides.

Ex: $2^x + 12 = 100$

$$2^x = 88$$

$$\ln 2^x = \ln 88$$

$$\frac{x \cancel{\ln 2}}{\cancel{\ln 2}} = \frac{\ln 88}{\ln 2}$$

$$x \approx$$

If you are solving an equation with the variable in the exponent AND down on the line of the problem, you can use graphing to solve.

Ex: $2^x + 12 = 10x$

Start by thinking about what the graphs of each side of the equation should look like and consider an appropriate window.

window: $\left\{ \begin{array}{l} x\text{-min} = 0 \\ x\text{-max} = 5 \\ x\text{-scl} = 1 \\ y\text{-min} = 0 \\ y\text{-max} = 20 \\ y\text{-scl} = 2 \end{array} \right.$

exponential graph up 12

line with a steep slope . . .

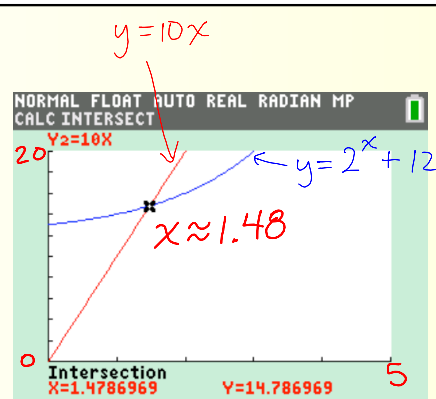
Ex: $2^x + 12 = 10x$

2. Enter the equations. Adjust window if necessary.

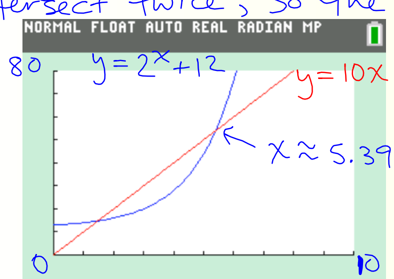
   y = 2^X + 12 and y = 10x.

3. Find the intersection(s) by going to your "calc" menu and choosing "intersect". Start on the left side of the intersection and set the cursor, then move to the right and set another mark, then enter.

$2^x + 12 = 10x$



Be Careful!
The graphs actually intersect twice, so the equation has 2 solutions:



$x \approx 1.48$ and

$x \approx 5.39$

Mixed AP and GP example:

Ann saves \$5 a week and currently has \$100.

100, 105, 110, ...
gives her balance at the beginning of nth week.

Gary account grows exponentially by 10%, and he starts with only \$20.

20, 22, 24.2,...
gives his balance at the beginning of nth week.

Will he ever catch up? When?

Solution:

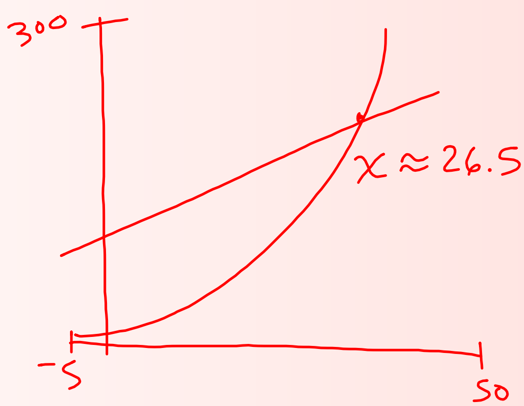
1 Ann	1 Gary
100, 105, 110, ...	20, 22, 24.2,...

$a_n = 5n + 95$

$g_n = 20(1.1)^{n-1}$

$$5n + 95 = 20(1.1)^{(n-1)}$$

window
$x(-5, 50)$
$xscl = 5$
$y(0, 300)$
$yscl = 20$



Groups:

A Geometric Sequence (GP) has the same first term as an Arithmetic Sequence (AP). The 3rd term of the GP is the same as the 10th term of the AP, and that term is 48. The 10th term of the AP is 4 times the 2nd term of the GP. Find d and r.

$g_1 = a_1 = u \qquad g_3 = a_{10} = 48 \qquad 48 = 4g_2$

$r = 4 \qquad d = 5$

Groups:

A Geometric Sequence (GP) has the same first term as an Arithmetic Sequence (AP). The 3rd term of the GP is the same as the 10th term of the AP, and that term is 48. The 10th term of the AP is 4 times the 2nd term of the GP. Find d and r.

Let $g_1 = a_1 = u \qquad g_3 = a_{10} = 48 \qquad a_{10} = 4g_2$

HW: SL Book

p. 262 #1- 7 (skip 4)

Show graph sketches when you use a graph. Don't forget scale.

Quiz Tuesday:
8.1.1
8.1.2
8.1.3
8.2.1
8.2.2

Group Quiz Wednesday:
Partial Fraction Decomposition